

# Class D spectral peak in Majorana quantum wires

Dmitry Bagrets and Alexander Altland<sup>1</sup>

<sup>1</sup>*Institut für Theoretische Physik, Universität zu Köln, Köln, 50937, Germany*

Proximity coupled spin-orbit quantum wires purportedly support midgap Majorana states at critical points. We show that in the presence of disorder these systems generate a second bandcenter anomaly, which is of different physical origin but shares key characteristics with the Majorana state: it is narrow in width, insensitive to magnetic fields, carries unit spectral weight, and is rigidly tied to the band center. Depending on the parity of the number of subgap quasiparticle states, a Majorana mode does or does not coexist with the impurity peak. The strong 'entanglement' between the two phenomena may hinder an unambiguous detection of the Majorana by spectroscopic techniques.

More than seven decades ago, the Majorana fermion was postulated as a neutral variant of the Dirac particle [1]. While the Majorana has never been observed in the canonical environments of particle physics, recent proposals[2–4] suggest a realization as a boundary state of one dimensional topological superconductors. The perspective to realize, and potentially apply topologically protected Majorana particles in condensed matter settings, has sparked a wave of theoretical [5–15] and experimental [16–19] activity.

Recently, three experimental groups have reported signatures of Majorana states in semiconductor quantum wires. In these experiments, proximity coupled semiconductor quantum wire purportedly hosting a Majorana particle are probed by tunneling spectroscopy. Evidence for the presence of the particle is drawn from the observation of a zero energy spectral peak. It has been verified that the presence of the peak hinges on parametric conditions necessary for the formation of a Majorana particle.

In this Letter we argue that the quasi one-dimensional quantum wires presently under consideration are prone to the formation of a *second* zero energy peak, caused by strong midgap quantum interference. The latter structure, here called (class D) spectral peak for brevity [20], shares key universal signatures with the Majorana peak, to the extent that disentangling the two phenomena may be difficult: it is (i) tied to zero energy, (ii) affected by temperature, but not (iii) by magnetic field, (iv) relies on parametric conditions similar to those required by the Majorana, (v) carries unit spectral weight (a single quasi 'state'), (vi) is independent of other midgap structures, such as Kondo resonances or Andreev bound states, shows (vii) sensitivity to the parity of channel number, however, unlike the Majorana, (viii) relies on the presence of moderately weak disorder.

*Qualitative discussion and main results.* — To start with, let us qualitatively discuss the physics underlying the spectral peak. The spin orbit and proximity coupled quantum wires presently under discussion form a variant of a spinless superconductor, more precisely a system of class D [21]. At the critical configuration separating a topologically trivial and nontrivial state (which will be the habitat of the Majorana fermion if parameters are

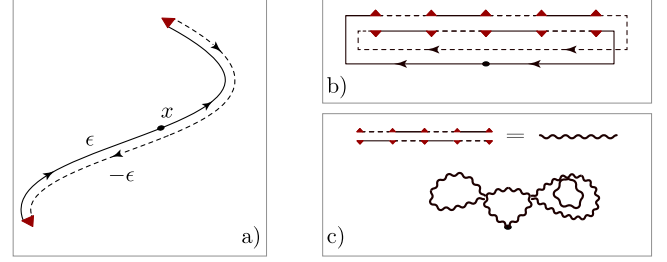


FIG. 1. a) Phase coherent scattering process  $p \rightarrow h \rightarrow p$ , b) low energy diffusion mode formed by succession of such processes, c) cartoon of a higher order quantum interference process involving self-interacting diffusion modes.

tuned in a space-like manner) the superconductor goes gapless. The presence of a Majorana particle signifies in a band center anomaly of the quasiparticle density of states (DoS). Quantum mechanically, the DoS at energy  $\epsilon$  (relative to the system's chemical potential,  $\mu$ ) is given by  $\rho(\epsilon) = -\frac{1}{\pi} \int dx \text{Im tr}(G^+(x, x; \epsilon))$ , where the Green function  $G^+(x, y; \epsilon) \equiv \langle x | (\epsilon + i0 + \mu \sigma_3^{\text{ph}} - \hat{H})^{-1} | y \rangle$ ,  $\sigma_3^{\text{ph}}$  is a Pauli matrix in the superconductor particle hole (ph) space, and 'tr' is a trace over that space. Semiclassically, the matrix elements  $G^{\text{pp}}(x, x; \epsilon)$  (or  $G^{\text{hh}}(x, x; \epsilon)$ ) may be interpreted as return amplitudes of quasi-particles, in an environment governed by the Bogoliubov-de Gennes Hamiltonian  $\hat{H}$  of the system. Much like in a gapless superconductor [22], the DoS in the proximity quantum wire is affected by the scattering of low energy excitations off spatial fluctuations of an order parameter amplitude,  $\Delta$ . To understand the physics of this mechanism [21], imagine a quasi-particle of energy  $\epsilon$  emanating from the point  $x$ . The particle has the option to scatter off the order parameter  $\Delta$  into a quasi-hole of energy  $-\epsilon$ , which by a second scattering process may get converted back to a particle returning to the point of origin. For low excitation energies, the particle and hole stretches involved in this process interfere constructively to give a robust (scattering phase insensitive) contribution to the return amplitude. Summation over repeated ph conversion processes generates an effective diffusion mode [21] similar to the diffuson and Cooperon modes of conventional disor-

dered metals. The increase of the effective quasi-particle gap away from the critical point confines the support of this mode to a small region in space – a disordered gapless superconductor ‘quantum dot’ – where it scales as  $\sim \delta/(2\pi\epsilon) \equiv s^{-1}$ ,  $\delta$  being the dot’s effective level spacing. In disordered electronic systems, singularities of this type are generally cut off by the nonlinear ‘self-interaction’ of diffusion modes (weak localization turning into strong localization, or level repulsion being prominent examples of such nonlinearities). In the present context, similar nonlinearities lead us to the result

$$\begin{aligned} N \text{ even : } \quad \frac{\rho(s)}{\rho_0} &= 1 + \frac{\sin(s)}{s}, \quad s = 2\pi \frac{\epsilon}{\delta} \\ N \text{ odd : } \quad \frac{\rho(s)}{\rho_0} &= 1 - \frac{\sin(s)}{s} + \delta \left( \frac{s}{2\pi} \right), \end{aligned} \quad (1)$$

where  $N$  is the number of channels in the system,  $\delta$  the average quasiparticle level spacing,  $\rho_0 \equiv \delta^{-1}$ , and the  $\delta$ -function is the contribution of the Majorana state. Multiple quantum interference multiplies the single diffusion mode by a phase factor  $\pm \sin(s)$ , where the overall sign depends on the channel number parity. The ensuing profiles of the band center DoS are shown in the inset to Fig. 2. Notice the superficially similar structures for even and odd channel numbers, while only in the latter case a genuine Majorana mode is present. For even channel numbers, a narrow width ( $\sim \delta$ ) peak is solely formed by class D diffusion modes. In either case, the anomalous contribution to the DoS integrates to the spectral weight  $\int d\epsilon (\rho(\epsilon) - \rho_0) = \frac{1}{2}$ . This means that tunneling spectroscopy limited in resolution to scales larger than  $\delta$  may not be able to unambiguously identify the Majorana state in situations where quantum interference is effective. An external loss of coherence, e.g. due to quasiparticle interactions, will suppress the class D peak, although a power law correction  $\sim |\epsilon|^{-1}$  to the density of states  $\rho(\epsilon)$  above an effective cutoff energy should remain visible. Finally notice how the phenomenon hinges on the presence of disorder: impurities render the low energy dynamics diffusive, and they provide the basis for the formation of an ‘impurity band’ of quasiparticles centered around the gap closing point in energy and space. However, we will argue below that even weak scattering  $\tau^{-1} \gtrsim \delta$  suffices to generate the effect. The rest of the paper is devoted to a derivation of Eq. (1). We will also discuss physical bounds on the spatial extension of the impurity quantum dot, the relevant disorder strengths, and other system parameters.

*Model.* — Majorana states can be realized in different variants of class D topological superconductors. For definiteness, we here consider the case of a proximity coupled helical liquid [11] subject to a smoothly varying magnetic field. However, we will argue below that our main conclusions are not rigidly tied to this setup. A helical liquid is formed by a system of left- and right-moving fermions,  $\psi_{L,\uparrow} \equiv \psi_L$  and  $\psi_{R,\downarrow} \equiv \psi_R$  carrying spin up

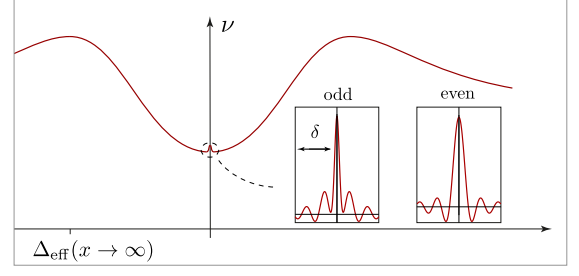


FIG. 2. Schematic profile of spectral density. The profile of the band center anomaly depends on the parity of the number of channels,  $N$ . Only for odd  $N$  the system supports a genuine Majorana level which (here represented as a broadened  $\delta$ -singularity.)

and down, respectively. (In the semiconductor setting, these modes form at the intersection of fermion bands shifted in momentum by strong spin-orbit interaction.) The coupling to a wire axis Zeeman field,  $B$ , and a proximity  $s$ -wave order parameter then generates the effective Hamiltonian  $\hat{H}_0 = \sum_{a=1}^N \int dx (\mathcal{H}_K^a + \mathcal{H}_B^a + \mathcal{H}_\Delta^a)$ , where  $\mathcal{H}_K^a = \sum_{C=L,R} \bar{\psi}_C^a i s_C v_F \partial_x \psi_C^a$ ,  $\mathcal{H}_B^a = B \bar{\psi}_L^a \psi_R^a + \text{h.c.}$ ,  $\mathcal{H}_\Delta^a = \Delta \bar{\psi}_L^a \psi_R^a + \text{h.c.}$ ,  $v_F$  is the Fermi energy, and  $s_C = (+/-)1$  for  $C = L/R$ . We have also introduced an index  $a = 1, \dots, N$ , which accounts for the option of multiple bands  $\psi_C^a$  crossing the Fermi energy. Differences in the Fermi velocities of the bands are absorbed in a rescaling of the fields. Ignoring the effects of band coupling by spin orbit interaction, the effects of static disorder and interaction, etc.  $\hat{H}_0$  has the status of a null theory describing the formation of a critical regime at  $B \simeq \Delta$ : introducing new state vectors (‘Majorana basis’),  $\eta^0 \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} -\psi_R - \bar{\psi}_R \\ i\psi_L - i\bar{\psi}_L \end{pmatrix}$ ,  $\eta^1 \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} i\psi_R - i\bar{\psi}_R \\ \psi_L + \bar{\psi}_L \end{pmatrix}$ , and assuming reality of the order parameter,  $\Delta \simeq B$ , the Hamiltonian  $\hat{H}_0$  assumes the form

$$\hat{H}_0 = \sum_{\mu=0,1} \int dx \eta^{\mu T} (-i v_F \partial_x \sigma_3 + (B - (-)^\mu \Delta) \sigma_2) \eta^\mu,$$

where a summation over channel indices is implicit and the Pauli matrices act in  $LR$ -space. Notice that the matrix structure sandwiched by the  $\eta$ ’s is antisymmetric, which is the defining condition of a class D (no symmetries other than particle-hole symmetry) superconductor. The above representation makes the formation of a low- ( $\eta^0$ ) and a high-energy sector ( $\eta^1$ ) manifest. In the absence of perturbations,  $\eta^{0,a}$  represent Majorana modes which go massless at the critical point  $\Delta_{\text{eff}} \equiv \Delta - B = 0$ . Throughout, we will assume a hierarchy of energy scales  $M \equiv B + \Delta \gg \Delta_{\text{eff}}$ , and on this basis we now turn to the discussion of perturbations,  $\delta \hat{H}$ , to  $\hat{H}_0$ . Scattering between the low- and the high-energy sector – generated by fluctuations of the chemical potential, or imaginary contributions to the order parameter – affects the low energy theory by contributions of  $\mathcal{O}(\delta H^2/M)$

and will be neglected throughout. By contrast, real fluctuations of the order parameter, or channel dependent differences in the field strength (indirectly generated by inter-channel spin-orbit coupling) couple within the low energy sector and have to be kept. We model such fluctuations in terms of a phenomenological ansatz  $\Delta_{\text{eff}} = \Delta_0(x) + W + V$ , where the first term describes a ramping of system parameters through a critical point at  $x = 0$ , where  $\Delta_0(0) = 0$  (cf. Fig. 2),  $W = \{W^{ab}\}$  is a matrix containing deterministic contributions to the inter-channel coupling, and  $V = \{V^{ab}(x)\}$  represents static disorder, here assumed to be Gaussian distributed,  $\langle V^{ab}(x)V^{a'b'}(x') \rangle = \delta^{aa'}\delta^{bb'}\delta(x-x')\gamma^2$ . Throughout we will limit our discussion to channels for which the low energy physics around  $x = 0$  is disorder dominated, i.e. for which the random potential  $V > W$  masks the deterministic contribution. (If this condition cannot be satisfied by a multiplet of channels, we are down to  $N = 1$ . [23])

*$\sigma$ -model.* — Our focus throughout will be on regions close to  $x = 0$  where the gap amplitude  $|\Delta(x)| < V$  is small enough for a class D impurity quantum dot to form. Such structures universally exhibit a zero energy spectral peak which we aim to explore quantitatively. To this end, we consider the supersymmetric functional integral  $\int D(\bar{\xi}, \xi) \exp(i \int dx \bar{\xi}(\epsilon^+ - \hat{H})\xi)$ , where  $\hat{H} = -iv_F \partial_x \sigma_3 - \Delta_{\text{eff}}(x)\sigma_2$  is the low energy ( $\mu = 0$ ) Hamiltonian, and  $\xi = \{\xi_{C,\alpha}\}$  and  $\bar{\xi} = \{\bar{\xi}_{C,\alpha}\}$  are integration variables with complex ( $\alpha = b$ ) and Grassmann valued ( $\alpha = f$ ) components. The DoS can be obtained from the above functional by differentiation w.r.t. suitably introduced sources, which we make not explicit for notational simplicity.

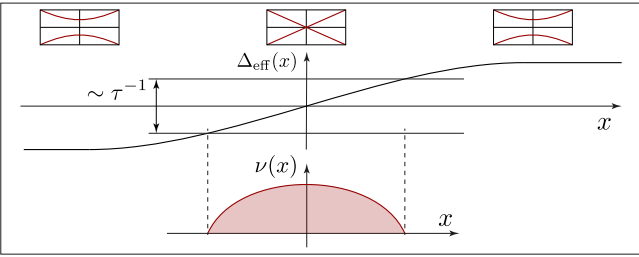


FIG. 3. Top: a sweep of an effective gap parameter generates a critical point at which the clean system goes gapless (cf. schematic dispersion relations shown in the insets.) Bottom: in the presence of disorder an impurity band of semicircular local average DoS forms.

We next average the generating functional over  $V$ , decouple the ensuing term quartic in  $\xi$  by Hubbard-Stratonovich transformation, and integrate over  $\xi$ , to obtain the action  $S[X, \bar{X}] = \frac{1}{2\gamma^2} \int dx \text{str}(X\bar{X}) + \frac{1}{2} \text{str} \ln(\epsilon^+ \tau_3 + X_S + X_A \sigma_3 - \hat{H}_0)$ , where 'str' is the supertrace [24] and  $\hat{H}_0 = \hat{H}|_{V=0}$ . Here we have introduced a new 2-component 'cc' space whose sectors  $\sim (\psi, \psi^T)$  contains states,  $\psi$ , and their Majorana-basis charge conju-

gates,  $\psi^T$ , resp. The Pauli-matrix,  $\tau_3$  acting in that space reflects the different transformation behavior of  $\hat{H}$  and  $\epsilon^+$ ,  $(\epsilon^+ - \hat{H})^T = -(\epsilon^+ - \hat{H})$ . The four component supermatrices  $X = \{X_{t\alpha, t'\alpha'}\}$ , and  $\bar{X} = X^\dagger \sigma_3^{\text{bf}}$  are Hubbard-Stratonovich fields, where the Pauli matrix in bf space ensures convergence,  $t, t'$  are indices in cc-space, and  $X_{S,A} = \frac{1}{2}(X \pm \bar{X})$ . Following standard strategies [24], we process the above action by a variational approach. Employing the polar ansatz,  $X = \bar{X} = i\lambda T \Lambda T^{-1}$ , where  $T$  are rotation matrices to be discussed momentarily and  $\Lambda$  is a diagonal matrix carrying real matrix elements  $\pm 1$ , we find that the modulus  $\lambda = \lambda(x) \geq 0$  is fixed by the variational equation  $\lambda(x) = \gamma^2 \text{Im}\langle x | (i\lambda - \hat{H}_0)^{-1} | x \rangle$ , which, upon substitution of  $\hat{H}_0$  becomes

$$\sum_a \int \frac{dp}{2\pi} \frac{\gamma^2}{(v_F p)^2 + \Delta^a + \lambda^2} = \sum_a \frac{\gamma^2}{v_F \sqrt{\Delta^a + \lambda^2}} = 1.$$

Here we have assumed the matrix  $\Delta_{\text{eff}}|_{V=0} \equiv \{\Delta^a \delta^{ab}\}$  diagonalized, and varying slowly enough in space for the functions  $\lambda(x)$  to adiabatically follow. Inspection of the equation shows that solutions exist if  $|\Delta^a| < N\gamma^2/v_F$  on average over  $a$ . In the limiting case of negligible channel dependence,  $\Delta^a \simeq \Delta_0$ , we readily obtain

$$\lambda(x) = \left( \left( \frac{1}{2\tau} \right)^2 - \Delta_0^2(x) \right)^{1/2},$$

where  $\tau^{-1} \equiv 2N\gamma^2/v_F$  is the golden rule scattering rate of the unperturbed spin-orbit quantum wire. The interpretation of  $G_0^+(\epsilon) \equiv (\epsilon^+ - \hat{H}_0 + i\lambda)^{-1}$  as an averaged Green function then yields  $\nu = 2\tau\nu_0\lambda$  for the (position dependent) DoS per unit length at saddle point level, where  $\nu_0 = N/(\pi v_F)$ . From this quantity, we generate the average DoS as  $\rho_0 = \int_{\nu(x)>0} dx \nu(x)$ , where the integral is over the region of support of the local DoS,  $|\Delta_0(x)| < 1/2\tau$  (cf. Fig. 3.) In passing we note that for large energies  $|\epsilon| \gg \tau^{-1}$ , the density of states assumes a form reminiscent of a smeared BCS profile to be discussed in more detail elsewhere [25].

Turning to the saddle point matrix  $\Lambda$ , we note that, as with the standard  $\sigma$ -model [24], the eigenvalue structure of the bosonic sector  $\Lambda_{bb} = \tau_3$  is fixed by the pole signature of the Green function. However, in the fermionic sector we have a choice,  $\Lambda_{ff} = \pm \tau_3$ , an ambiguity intimately related to the formation of a Majorana peak [26]: in the limit of low energies,  $\epsilon \rightarrow 0$ , spatially uniform rotations  $\Lambda \rightarrow T \Lambda T^{-1} \equiv Q$  leave the action invariant. The symmetries of our class D theory require [27]  $T_{bb} \in \text{Sp}(2)/\text{U}(1)$ , while  $T_{ff} \in \text{O}(2)/\text{U}(1) \simeq \mathbb{Z}_2$ . This latter identity means that in the fermionic sector we have only two discrete symmetry transformations,  $T_{ff} = \mathbb{1}$ , and  $T_{ff} = \tau_1$ , where the latter acts by an exchange  $T_{ff} \tau_3 T_{ff}^{-1} = -\tau_3$  of the diagonal saddle points. There are no continuous transformations  $T$  interpolating between  $\Lambda_1$  and  $\Lambda_2$ , which means that the saddle point manifold,  $\mathcal{M} \equiv \mathcal{M}_1 \cup \mathcal{M}_2$

splits into two disjoint components, where  $\mathcal{M}_s$ ,  $s = 1, 2$  is generated by action of all transformations  $T$  with  $T_{\text{ff}} = \mathbb{1}$  on the saddle point  $\Lambda = \Lambda_1 \equiv \tau_3$  and  $\Lambda = \Lambda_2 \equiv \tau_3 \otimes \sigma_3^{\text{bf}}$ , resp.

We finally substitute slowly fluctuating configurations  $X \rightarrow i\lambda Q$  into the action and expand in small symmetry breaking contributions  $\sim \epsilon^+ Q$  and  $\partial Q$ . The expansion follows standard procedures [22, 24] and generates the effective action

$$S[Q] = \int dx \frac{\pi\nu(x)}{8} \text{tr} (D(x) \partial Q \partial Q + 4\epsilon\tau_3 Q) + S_{\text{par}}[Q],$$

where  $S_{\text{par}}[Q] = \frac{1}{2} \text{str} \ln(\Lambda - \hat{H}_0)$  is the Pfaffian of the model Hamiltonian at saddle point level, and  $D = \frac{v_F}{\pi\nu} \sum_a \frac{\lambda^2}{(\lambda^2 + \Delta_a^2)^{3/2}}$  plays the role of an effective diffusion coefficient, which in the isotropic limit  $\Delta^a \simeq \Delta_0$  simplifies to  $D \sim (v_F\tau)^2 \lambda$ . Notice how in the disorder dominates regions  $|\Delta| \ll \lambda$ ,  $D \sim v_F^2 \tau$  reduces to the diffusion constant of a metallic system, while  $D \rightarrow 0$  at the boundaries  $\lambda \rightarrow 0$  of the metallic domain. The structure of the action above indicates that fluctuations of  $Q$  generate the diffusive soft modes discussed in connection with Fig. 1.

*Quasiparticle 'quantum dot'.* — For energies  $\epsilon \lesssim \langle D \rangle / L^2$  lower than the minimum excitation energy of spatially varying  $Q$ -fluctuations ( $L$  is the extension of the metallic domain  $\lambda > |\Delta_0|$ , and  $\langle D \rangle$  is the typical value of the diffusion coefficient), the action is dominated by homogeneous modes  $Q = \text{const}$ . In this regime, the system truly behaves like a 'quantum dot', and the action collapses to

$$S[Q] = \frac{\pi\epsilon^+}{2\delta} \text{tr}(Q\tau_3) + S_{\text{par}}[Q]. \quad (2)$$

Here,  $S_{\text{par}}$  is a topological term [26] which in the present context assumes the form  $S_{\text{par}}[Q] = S_{\text{par}}[\Lambda_1] = 0$  and  $S_{\text{par}}[Q] = S_{\text{par}}[\Lambda_2] = \text{tr} \ln(G_0^+(0) - G_0^-(0))$  for  $Q \in \mathcal{M}_{1,2}$  resp. Using the particle-hole symmetry of the spectrum, the latter expression evaluates to  $S_{\text{par}}[\Lambda_2] = -i\pi\mathcal{N}_{\text{tot}}$  where  $\mathcal{N}_{\text{tot}} = \sum_a N_a$ , and  $N_a$  is the total number of states carried by mode  $a$ . While this number does not carry physical meaning by itself,  $\exp(-S_{\text{par}}) = (-)^{\mathcal{N}_{\text{tot}}}$  responds only to the *parity* of  $\mathcal{N}_{\text{tot}}$ . Now, robust arguments show that  $N_a = (2k+1)$  is generally odd [28]. This means that  $(-)^{\mathcal{N}_{\text{tot}}} = (-)^N$  is susceptible to the parity of the number of channels. The DoS resulting from Eq. (2), computed in Ref. [26] by non-perturbative integration over  $\mathcal{M}_{1,2}$ , is given by Eq. (1).

While Eq. (1) has been derived for a specific model, the high degree of universality of the result — dependence on no parameters other than the ratio  $\epsilon/\delta$  — testifies to wider applicability. Generally speaking, we expect the result to hold provided  $\delta \lesssim \tau^{-1} \ll \Delta$ , i.e. the randomness hybridizing a number of levels of the clean system around the critical point  $x \simeq 0$ . This expectation is corroborated by the observation of similar spectral profiles for phenomenological random matrix models [21].

(A straightforward numerical calculation indeed shows that peaks similar to (1) form already for *two* randomly coupled levels.) Finally, random spectra are notorious for their 'level rigidity', i.e. the weakness of sample-to-sample fluctuations. Signatures of the anomaly in the *ensemble averaged global DoS* are therefore expected to show in both the sample specific global and local DoS.

Summarizing, we have shown that in the presence of even weak disorder, a Majorana peak in class D quantum wires will coexist with a spectral anomaly. The two phenomena share key universal features, are of comparable magnitude, and integrate to a spectral weight insensitive to the parity of the channel number. All this makes us speculate that disorder may be detrimental to an unambiguous observation of the Majorana particle by spectroscopic means.

We thank F. von Oppen for discussions. Work supported by SFB/TR12 of the Deutsche Forschungsgemeinschaft.

- 
- [1] E. Majorana, *Nuovo Cimento* **14**, 171 (1937).
  - [2] A. Kitaev, *Phys. Usp.* **44**, 131 (2001).
  - [3] Y. Oreg, G. Refael, and F. von Oppen, *Phys. Rev. Lett.* **105**, 177002 (2010).
  - [4] R. M. Lutchyn, J. D. Sau, and S. Das Sarma, *Phys. Rev. Lett.* **105**, 077001 (2010).
  - [5] J. Alicea, *Reports on Progress in Physics* **75**, 076501 (2012).
  - [6] K. Flensberg, *Phys. Rev. B* **82**, 180516 (2010).
  - [7] A. Alicea, Y. Oreg, and F. von Oppen, *Nature Physics* **7**, 412 (2011).
  - [8] L. Fidkowski, R. M. Lutchyn, C. Nayak, and M. P. A. Fisher, *Phys. Rev. B* **84**, 195436 (2011).
  - [9] E. M. Stoudenmire, J. Alicea, O. A. Starykh, and M. P. Fisher, *Phys. Rev. B* **84**, 014503 (2011).
  - [10] R. M. Lutchyn, T. D. Stanescu, and S. Das Sarma, *Phys. Rev. Lett.* **106**, 127001 (2011).
  - [11] E. Sela, A. Altland, and A. Rosch, *Phys. Rev. B* **84**, 085114 (2011).
  - [12] A. R. Akhmerov, J. P. Dahlhaus, F. Hassler, M. Wimmer, and C. W. J. Beenakker, *Phys. Rev. Lett.* **106**, 057001 (2011).
  - [13] P. W. Brouwer, M. Duckheim, A. Romito, and F. von Oppen, *Phys. Rev. Lett.* **107**, 196804 (2011).
  - [14] G. Kells, D. Meidan, and P. W. Brouwer, *Phys. Rev. B* **85**, 060507 (2012).
  - [15] B. van Heck, A. R. Akhmerov, F. Hassler, M. Burrello, and C. W. J. Beenakker, *New Journal of Physics* **14**, 035019 (2012).
  - [16] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, *Science* **336**, 1003 (2012).
  - [17] L. Rokhinson, X. Liu, and J. Furdyna, *arXiv:1204.4212* (2012).
  - [18] M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caroff, and H. Q. Xu, *arXiv:1204.4130* (2012).
  - [19] A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, and H. Shtrikman, *arXiv:1205.7073* (2012).

- [20] Strictly speaking, the system is in class D or B depending on whether a Majorana is absent or present. We stick to the designation 'D' for simplicity.
- [21] A. Altland and M. R. Zirnbauer, Phys. Rev. B **55**, 1142 (1997).
- [22] A. Altland, B. D. Simons, and D. Taras-Semchuk, Advances in Physics **49**, 321 (2000).
- [23]  $N = 1$  is a special case. For abruptly varying control parameters (an 'interface'), the system then supports only one level below the bulk gap, and our analysis does not apply. For smoothly varying parameters Anderson localization on length scales  $l \sim \tau v_F$  may become an issue. Although the detailed analysis is beyond the scope of this paper, a band center peak should be observable also in this case.
- [24] K. Efetov, *Symmetry in disorder and chaos* (Cambridge University Press, Cambridge, 1997).
- [25] D. Bagrets and A. Altland to be published.
- [26] D. A. Ivanov, Journal of Mathematical Physics **43**, 126 (2002).
- [27] M. R. Zirnbauer, Journal of Mathematical Physics **37**, 4986 (1996).
- [28] The point is that each mode,  $a$ , is described by an Hamilton operator obeying the symmetry  $[\hat{H}, \sigma_1]_+ = 0$ . This chirality implies that all eigenstates of non-vanishing energy come in degenerate pairs. For gap functions  $\Delta_{\text{eff}}$  possessing a critical point,  $\Delta_{\text{eff}}(0) = 0$ , the channel supports a non-degenerate zero energy state, i.e. the total number of states,  $N_a$ , is odd.